$$\frac{1}{2}\begin{vmatrix} a_2, & b_2, & a_2, & b_2 \\ a_2, & b_3, & a_3, & b_3 \\ a_4, & b_4, & a_4, & b_4 \\ a_5, & b_5, & a_5, & b_5 \end{vmatrix} \equiv (23)(45)-(24)(35)+(25)(34)\equiv 0,$$

which cannot be satisfied by

$$(23)=(45)=(24)=(35)=(25)=(34).$$

The general case fails in consequence of a similar identical relation among the determinants of a matrix. This relation can be expressed as the expansion of a determinant of $2(n-3)$ rows which can be symbolized by $\frac{A \mid C}{B \mid C}$, where A, B, C denote, respectively,

$$\begin{vmatrix} 0, & 0, & \ldots, & a_{n-2} \\ 0, & 0, & \ldots, & b_{n-2} \\ \ldots & \ldots & & \ldots \\ 0, & 0, & \ldots, & l_{n-2} \end{vmatrix}, \quad \begin{vmatrix} a_2, & a_3, & a_4, & \ldots, & a_{n-2} \\ b_2, & b_3, & b_4, & \ldots, & b_{n-2} \\ \ldots & \ldots & \ldots & & \ldots \\ l_2, & l_3, & l_4, & \ldots, & l_{n-2} \end{vmatrix}, \quad \begin{vmatrix} a_{n-1}, & a_n, & a_2, & a_3, & \ldots, & a_{n-4} \\ b_{n-1}, & b_n, & b_2, & b_3, & \ldots, & b_{n-4} \\ \ldots & \ldots & \ldots & \ldots & & \ldots \\ l_{n-1}, & l_n, & l_2, & l_3, & \ldots, & l_{n-4} \end{vmatrix}.$$

When this is expanded according to Laplace's method in determinants of $(n-3)$ rows, we get

$$\begin{aligned} &(2, 3, \ldots\ldots, n-3, n-2)(n-1, n, 2, \ldots\ldots, n-4) \\ -&(2, 3, \ldots\ldots, n-3, n-1)(n-2, n, 2, \ldots\ldots, n-4) \\ +&(2, 3, \ldots\ldots\ldots\ldots, n-3, n)(n-3, n, 2, \ldots\ldots, n-4)=0, \end{aligned}$$

which cannot be satisfied if these determinants are all equal.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Problem 206 was also solved by J. E. Sanders; No. 207 was also solved by L. E. Newcomb.

208. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve (1) .... $x^4+y^4=14x^2y^2$; (2) .... $x+y=m$.

I. Solution by EDWIN L. RICH, Student at Lehigh University.

Equation (1) may be written

$$(x^2-y^2-xy\sqrt{12})(x^2-y^2+xy\sqrt{12})=0 \ldots\ldots (3).$$

Solving each factor,

$$x=(\sqrt{3}\pm 2)y \text{..........} (4),$$
$$x=(-\sqrt{3}\pm 2)y \text{..........} (5).$$

From the simultaneous equations (2), (4) and (2), (5),

$$x=\frac{m}{6}(3\pm\sqrt{3}), \quad y=\frac{m}{6}(3\mp\sqrt{3}),$$

$$x=\frac{m}{2}(1\pm\sqrt{3}), \quad y=\frac{m}{2}(1\mp\sqrt{3}).$$

II. **Solution by S. A. COREY, Hiteman, Iowa.**

Adding $2x^2y^2$ to each member of (1),

$$x^4+2x^2y^2+y^4=16x^2y^2 \text{..........} (3),$$

whence $x^2+y^2=\pm 4xy$ .... (4).

Squaring (2), we have

$$x^2+2xy+y^2=m^2 \text{..........} (5);$$

substituting from (4), $6xy=m^2$, or $-2xy=m^2$,

whence, $x=\dfrac{m^2}{6y}$ or $-\dfrac{m^2}{2y}$.

By substituting these values of x in (2), we find without difficulty,

$$x=m(\tfrac{1}{2}\pm\sqrt{\tfrac{1}{12}}), \text{ or } m(\tfrac{1}{2}\pm\sqrt{3/4});$$
$$y=m(\tfrac{1}{2}\mp\sqrt{\tfrac{1}{12}}), \text{ or } m(\tfrac{1}{2}\mp\sqrt{3/4}).$$

Also solved by R. L. Borger, G. W. Greenwood, E. L. Sherwood, L. E. Newcomb, J. F. Lawrence, S. E. Harwood, Elmer Schuyler. G. B. M. Zerr, and J. Scheffer.

209. **Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.**

Prove that $(a^4+b^4+c^4+d^4)>4abcd$.

Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, O.

$(a^2+b^2)>2ab$ and $(c^2+d^2)>2cd$ from which, by multiplication, $(a^2c^2+b^2c^2+a^2d^2+b^2d^2)>4abcd$.......... (1).

Again $(a^4+c^4)>2a^2c^2$; $(b^4+c^4)>2b^2c^2$; $(a^4+d^4)>2a^2d^2$, and $(b^4+d^4)>2b^2d^2$.

By addition, $(a^4+b^4+c^4+d^4)>(a^2c^2+b^2c^2+a^2d^2+b^2d^2)$ and this in connection with (1) gives $(a^4+b^4+c^4+d^4)>4abcd$.

Also solved by G. W. Greenwood, R. L. Borger, E. L. Sherwood, L. E. Newcomb, Elmer Schuyler, J. H. Meyer, A. J. Haun, O. A. Laisant, J. Scheffer.